1–2 percent. This accuracy was obtained in all cases, where the resonances could be excited and detected.

SUMMARY

It has been shown that the accuracy of calculating the resonance frequencies of circular and rectangular microstrip disk resonators can be improved significantly, if a new resonator model is defined. The resonator model takes into consideration the influence of the electrical and magnetic stray field at the edges of the resonator on the effective dimensions of the resonator and the influence of the fringing field as well as the influence of the inhomogeneous field distribution of the different resonance modes on the effective dielectric constant. Using the new defined resonator models, the accuracy for calculating the resonance frequencies is about 1 percent. This result is about five to ten times better than the results of calculating methods known in the literature previously.

REFERENCES

[1] M. Caulton, B. Hershenov, S. P. Knight, and R. E. DeBrecht, "Status of lumped elements in microwave integrated circuits—Present and future," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Integrated Circuits*), vol. MTT-19, pp. 588–599, July 1971.
[2] C. S. Aitchison *et al.*, "Lumped-circuit elements at microwave frequencies," *IEEE Trans. Microwave Theory Tech.* (*1971 Symposium Issue*), vol. MTT-19, pp. 928–937, Dec. 1971.
[3] A. Farrar and A. T. Adams, "Matrix methods for microstrip three-dimensional problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 497–504, Aug. 1972.
[4] P. Benedek and P. Silvester, "Capacitance of parallel rectangular plates separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 504–510, Aug. 1972.
[5] T. Itoh and R. Mittra, "A new method for calculating the capacitance of a circular disk for microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 431–432, June 1973.
[6] S. Mao, S. Jones, and G. D. Vendelin, "Millimeter-wave integrated circuits," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Integrated Circuits*), vol. MTT-16, pp. 455–461, July 1968.
[7] J. G. Kretzschmar, "Theoretical results for the elliptic microstrip resonator," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-20, pp. 342–343, May 1972.
[8] I. Wolff and N. Knoppik, "The microstrip ring resonator and dispersion measurements on microstrip lines," *Electron. Lett.*, vol. 7, pp. 779–781, Dec. 1971.
[9] Y. S. Wu and F. J. Rosenbaum, "Mode chart for microstrip ring resonators," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 487–489, July 1973.
[10] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[11] M. V. Schneider, "Microstrip lines for microwave integrated circuits," *Bell Syst. Tech. J.*, vol. 48, pp. 1421–1444, May 1969.
[12] G. Kirchhoff, *Gesammelte Abhandlungen.* Leipzig, Germany: 1882, pp. 101–113.
[13] O. Zinke, *Widerstände, Kondensatoren, Spulen und ihre Werkstoffe.* Berlin, Germany: Springer, 1965, p. 82.
[14] J. Watkins, "Circular resonant structures in microstrip," *Electron. Lett.*, vol. 5, pp. 524–525, Oct. 16, 1969.
[15] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30–39, Jan. 1971.
[16] H. Hofmann, "Dispersion of the ferrite-filled microstrip-line," *Arch. Elek. Übertragung.*, vol. 28, pp. 223–227, May 1974.

Hybrid Branchline Couplers—A Useful New Class of Directional Couplers

BURKHARD SCHIEK

Abstract—**The hybrid branchline coupler consists of two transmission lines connected alternately by $\lambda/4$ shunt and series branches. The analysis of this structure leads to a class of directional couplers of which the parallel transmission-line and the de Ronde strip-slot types may be regarded as special cases. From the precise design data thus available, a number of 3-dB strip-slot couplers have been built in C band and X band with a performance close to the predicted one.**

I. INTRODUCTION

For the design of microwave systems with integrated circuitry the broad-band 90° 3-dB coupler is a particularly important structure and a considerable effort has been devoted to the problem of realizing such a coupler in a planar form.

Among the structures commonly used is the branchline coupler with two or three branches. This type of coupler is realizable in planar microstrip technique for a wide range of frequencies but the useful bandwidth is limited. The branchline coupler with four branches has a somewhat larger bandwidth but the impedance values of the outer branches are impractical.

Recently, the Lange coupler [1] has found wide interest but because of the necessary bonding wires the coupler is not really planar and therefore a realization at very high frequencies is difficult. Wide-band 3-dB coupling is mostly realized with the parallel-coupled transmission-line directional coupler [2]. This type of coupling is particularly suited for a triplate technique which in

Manuscript received January 21, 1974, revised March 13, 1974.
The author is with the Philips Forschungslaboratorium Hamburg GmbH, Hamburg, Germany.

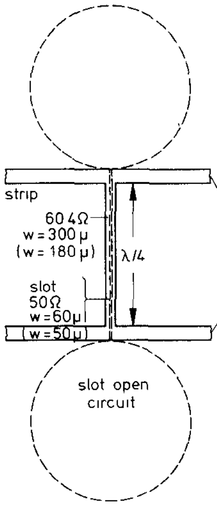

Fig. 1. De Ronde's strip-slot coupler in C band. Substrate thickness: 0.63 mm; in parenthesis the dimensions of the X-band coupler with 0.38-mm substrate.

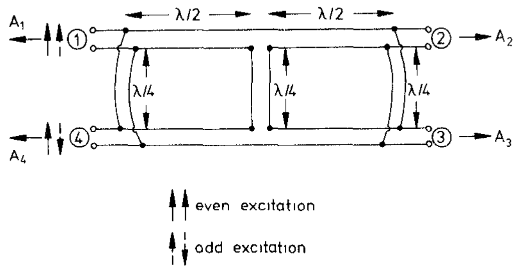

Fig. 2. Hybrid branchline coupler with branches alternately shunt and series connected in a two-wire presentation. ↑↑: Even excitation. ↑↓: Odd excitation.

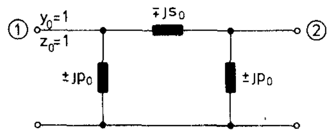

Fig. 3. Even/odd mode excitation with $\lambda/4$ branches. s_0: Normalized wave impedance. p_0: Normalized wave admittance.

addition allows the realization of geometrical backward as well as forward couplers. An outstanding feature of the transmission-line coupler is the fact that matching and directivity may be perfect, independent of frequency, at least under ideal conditions. Unfortunately, 3-dB coupling is very difficult to achieve in planar microstrip technique with normal chemical etching, although 5–6-dB coupling can be realized.

De Ronde has suggested [3] a somewhat different structure (Fig. 1) with a strip on the upper side and a slot in the ground plane of a ceramic substrate. With this strip-slot coupler 3-dB coupling can easily be achieved and the structure is planar, although both sides of the substrate have to be etched. A handicap for the acceptance of this coupler has been the fact, among other reasons, that no simple design theory is available, although the data of an empirical optimization have been published [4]. The purpose of this paper is to present a design theory for de Ronde's strip-slot coupler. From this theory 3-dB couplers have been built at C band and X band with a measured performance very close to the expected behavior. The theory to be presented will show that de Ronde's strip-slot coupler is just a special case within a whole family of couplers, which we name "hybrid branchline couplers," that all resemble the parallel transmission-line coupler.

II. THEORY OF THE HYBRID BRANCHLINE COUPLER

It is well known that a branchline coupler with $\lambda/4$ shunt arms needs a spacing of the shunt arms of a quarter wavelength in order to arrive at a useful coupler. Let us now assume that the branches are alternately shunt and series connected to the through-going lines as shown in Fig. 2. Let the normalized shunt arm wave admittance be p_0 and the normalized series wave impedance be s_0. It turns out that a useful backward coupler can be obtained with a spacing of the branches of half a wavelength. This coupler, however, is not particularly broad band. But of course, a transmission line half a wavelength long can be omitted and it results in a branchline coupler with only two junctions. We shall name this a hybrid branchline coupler, because the branches are alternately shunt and series connected.

An even $(+)$ and odd $(-)$ mode excitation leads to two two-port networks which are sketched in Fig. 3. The further analysis can follow in all details the method of Reed and Wheeler [5]. With A_1, A_2, A_3, A_4 as the vector amplitudes of the signals emerging from the four arms (Fig. 2), and with the $ABCD$-matrix $M_\pm$ for the even $(+)$ and odd $(-)$ excitations

$$M_\pm = \begin{bmatrix} A & \pm jB \\ \pm jC & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \pm jp_0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \mp js_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \pm jp_0 & 1 \end{bmatrix} = \begin{bmatrix} 1 + p_0 s_0 & \mp js_0 \\ \pm jp_0[2 + p_0 s_0] & 1 + p_0 s_0 \end{bmatrix} \tag{1}$$

all relevant quantities can be calculated [5]. The quantities p_0, s_0 are dimensionless and positive.

Perfect match and directivity, i.e., $A_1 = A_3 = 0$, requires $B = -C$. A 3-dB coupling is given by $|B| = 1$. This leads to the equations

$$s_0 = 2p_0 + p_0^2 \cdot s_0 \tag{2a}$$

and

$$s_0 = 1 \tag{2b}$$

from which one obtains

$$s_0 = 1 \quad \text{(normalized wave impedance)}$$
$$p_0 = \sqrt{2} - 2 \quad \text{(normalized wave admittance)} \tag{3}$$

or

$$s_0 \mathrel{\hat{=}} 50\ \Omega \text{ and}$$

$$p_0 \cong 1/120.8\,\Omega$$

in a 50-Ω system.

For an arbitrary coupling

$$|A_2|^2 = \frac{1}{A^2} = \left[\frac{1}{1+p_0 s_0}\right]^2 = \left[\frac{1-p_0^2}{1+p_0^2}\right]^2. \qquad (4)$$

After p_0 has been determined from the required coupling, s_0 follows from (2a).

The necessary series branch may be realized by a slotline in the conducting ground plane of a ceramic substrate and the shunt branches by microstrips. Then a circuit results as shown in Fig. 4. The coupling only takes place in the junction area, the three branches themselves are supposed to be free of coupling. Once the frequency dispersion of the branchlines is known it is but little work to calculate the frequency dependence of the coupling. A particularly simple result is obtained if the phase characteristic of the three branches is equal to that of a pure TEM mode, and with all branches being $\lambda/4$ at the center frequency. Then the reactances $\mp js_0$, $\pm jp_0$ of Fig. 3 for the even (+) and odd (−) excitation as a function of the frequency deviation $\Delta f = f - f_0$ from the center frequency f_0 have to be substituted by

$$\text{excitation}\begin{cases}\text{even } (+) \quad -js_0 \rightarrow -js_0 \cdot \cot x & \quad jp_0 \rightarrow jp_0 \cdot \tan x \\ \text{odd } (-) \quad +js_0 \rightarrow js_0 \tan x & \quad -jp_0 \rightarrow -jp_0 \cot x\end{cases} \qquad x = 45^\circ\left(1 + \frac{\Delta f}{f_0}\right). \qquad (5)$$

With (5) and a straightforward calculation [5] it turns out that

$$A_1 = A_3 = 0 \qquad (6a)$$

independent of the frequency and

$$A_2 = \frac{2}{2(1+p_0 s_0) + js_0[\tan x - \cot x]} \qquad (6b)$$

$$A_4 = -j \cdot \frac{s_0[\tan x + \cot x]}{2(1+p_0 s_0) + js_0[\tan x - \cot x]}. \qquad (6c)$$

Note that in [5] the phase at the end of a $\lambda/4$ line section is defined as $-j$.

The equations (6) give exactly the same frequency dependency as the ideal transmission-line coupler, as given, e.g., in [6, eq. (18)–(23)] and keeping in mind that $2x = \theta$ and with $\theta = 90°$ at the center frequency as defined in [6].

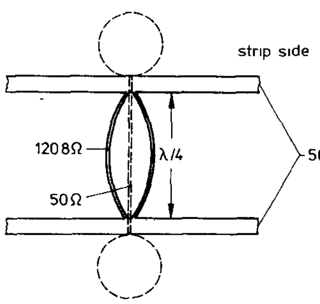

Fig. 4. Strip-slot-strip 3-dB hybrid branchline coupler.

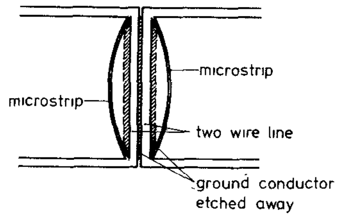

Fig. 5. Hybrid branchline coupler realized with a two-wire line as the series branch.

In general, the electrical length of the branches may be equal and equivalent to $\lambda/4$ at the center frequency. However, due to the different dispersion characteristics of stripline and slotline this equality of electrical length may not be maintained at other frequencies.

III. EQUIVALENCE OF THE HYBRID BRANCHLINE COUPLER AND THE PARALLEL-TRANSMISSION-LINE COUPLER

In Fig. 4 the series-connected branch was realized by a slotline. It is, of course, also possible to use a two-wire transmission line, i.e., two parallel strips with the ground conductor etched away. The two shunt branches are realized by two additional microstrips as shown in Fig. 5.

In the case of the parallel-coupled transmission-line coupler the two parallel conductors take over the function of the series branch as well as the shunt branches in the form of a linear superposition. Commonly, the transmission-line coupler is described in terms of the even and odd mode normalized wave impedance z_{0e} and z_{0o}. It is easy to show that the hybrid branchline coupler of Fig. 3 or Fig. 4 is perfectly equivalent to the transmission-line coupler, if

$$\frac{1}{z_{0o}} - \frac{1}{z_{0e}} = \frac{2}{s_0}$$

$$z_{0e} = \frac{1}{p_0}. \qquad (7)$$

It is therefore correct to say that the transmission-line coupler is a special member of the family of hybrid branchline couplers.

IV. OTHER STRUCTURES WHICH BELONG TO THE CLASS OF HYBRID BRANCHLINE COUPLERS

1) The 3-dB coupler shown in Fig. 4 uses two microstrips with 120.8 Ω. These two strips can be combined to one strip with 60.4 Ω, parallel and on top of the 50-Ω slot (Fig. 1). This is the coupler as suggested by de Ronde. Experimental results obtained for this coupler will be reported in the following.

2) So far we have discussed a hybrid branchline coupler

in a strip-slot-strip or π-configuration (Fig. 3). Of course, a slot-strip-slot or T-configuration (Fig. 6) is also possible. Perfect match and directivity requires

$$s_1[2 + s_1 \cdot p_1] = p_1 \qquad (8)$$

and the coupling is given by

$$|A_2|^2 = \frac{1}{A^2} = \left[\frac{1}{1 + s_1 p_1}\right]^2 = \left[\frac{1 - s_1^2}{1 + s_1^2}\right]^2. \qquad (9)$$

Thus 3-dB coupling or $|A_2|^2 = 1/2$ leads to

$$p_1 = 1 \qquad \text{(normalized wave admittance)}$$

$$s_1 = \sqrt{2} - 1 \qquad \text{(normalized wave impedance)}$$

or $p_1 \mathrel{\hat{=}} 1/50\ \Omega$ and $s_1 \mathrel{\hat{=}} 20.7\ \Omega$ in a 50-Ω system.

This type of coupler has also been realized and will be discussed in the following. The theoretical frequency behavior of the slot-strip-slot coupler with TEM lines is again identical to the one section transmission-line coupler.

3) The number of branches may be further increased. The analysis is very simple, if we connect identical structures in tandem as shown in Fig. 7 for a strip-slot-strip configuration. For 3-dB coupling, match, and perfect directivity, one obtains:

$$2s_2[1 + p_2 s_2] = 1$$

and

$$s_2 = p_2[2 + p_2 s_2]. \qquad (10)$$

Equations (10) have the solutions $s_2 = 0.455$ as a normalized wave impedance and $p_2 = 0.217$ as a normalized wave admittance (Fig. 7), or $s_2 \mathrel{\hat{=}} 22.7\ \Omega$ and $p_2 = 1/230\ \Omega$ in a 50-Ω system. These values are difficult to realize. The theoretical frequency characteristic is again the same as for the transmission-line coupler. The inner two strips can be combined to one.

4) It is noteworthy that an identical coupler to the hybrid branchline coupler in π- or T-configuration can be obtained with only one series branch, one shunt branch, and an ideal transformer as shown in Fig. 8. This is also known as the Norton transformation. The values of the elements $\tilde{s}_0$, $\tilde{p}_0$, n^2 can easily be calculated for match and perfect directivity and any given coupling, however, they are not given here. Of course, also a mixture of π- or T-configuration and a transformer is conceivable.

5) So far all the coupler types discussed were backward couplers. Forward coupling may be obtained, if either the series or shunt branches of the hybrid branchline coupler have a length of $3/4\ \lambda$ or a length of $\lambda/4$ plus phase reversal. For a π-type configuration as shown in Fig. 9, perfect match and directivity and 3-dB coupling leads to

$$p_3 = \sqrt{2} - 1 \qquad s_3 = \frac{\sqrt{2}}{2}. \qquad (11)$$

Unfortunately, for this forward coupler matching and directivity are no longer perfectly independent of frequency. But assuming that the circuit of Fig. 9 can be realized by $\lambda/4$ strips and a $\lambda/4$ slot with additional frequency independent phase reversal, a useful bandwidth of 60 percent has been calculated. Within this bandwidth, directivity was better then 20 dB and coupling within 3 dB $\pm$ 1 dB. But, of course, a simple broad-band phase reversal network has yet to be found.

6) In the strip-slot-strip coupler of Fig. 4 the slots are reactively terminated by a high impedance. One can also design a coupler where the slotlines lead to output ports while two of the striplines are reactively terminated by a high impedance as shown in Fig. 10. It is quite easy to give the appropriate dimension, e.g., for 3-dB coupling (Fig. 10), as the role of the series branch and one shunt branch interchanges. The two $\lambda/4$ strips may be combined to one but with the slot on the bottom displaced to the center of the microstrip. Such a coupler may be used as a

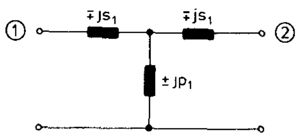

Fig. 6. Slot-strip-slot coupler or T-configuration.

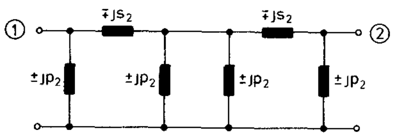

Fig. 7. Strip-slot-strip-slot-strip coupler.

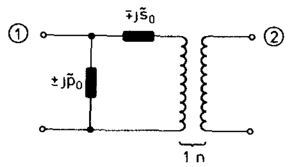

Fig. 8. Strip-slot transformer coupler.

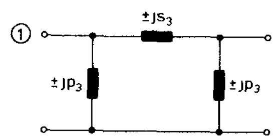

Fig. 9. Forward coupler in π-configuration.

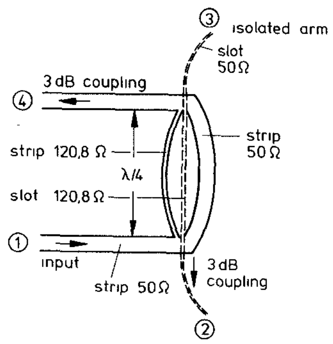

Fig. 10. 3-dB coupler with strip and slot ports.

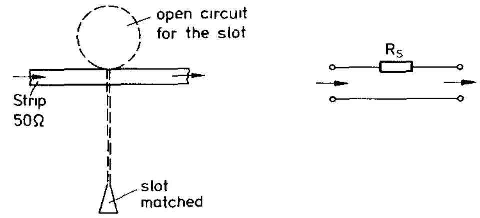

Fig. 11. Structure for the measurement of the equivalent series impedance R_s for a slot, crossing a microstrip.

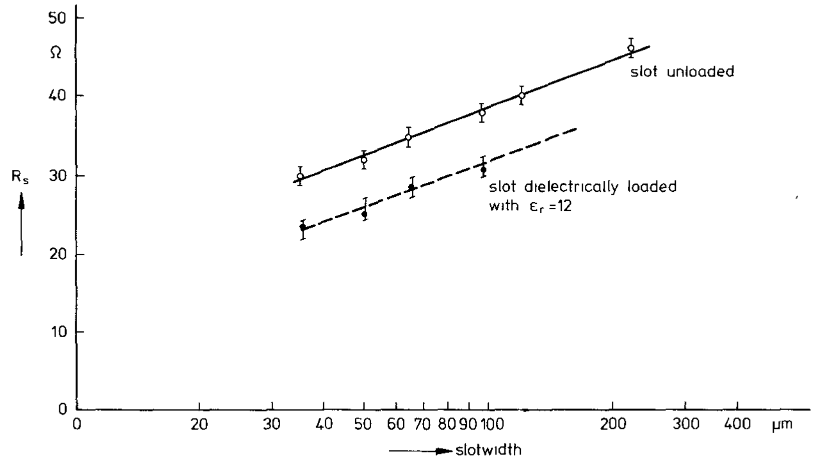

Fig. 12. Equivalent series impedance R_s versus slot width. Frequency of measurement C band. Substrate thickness: 0.63 mm. $\epsilon_r = 12$.

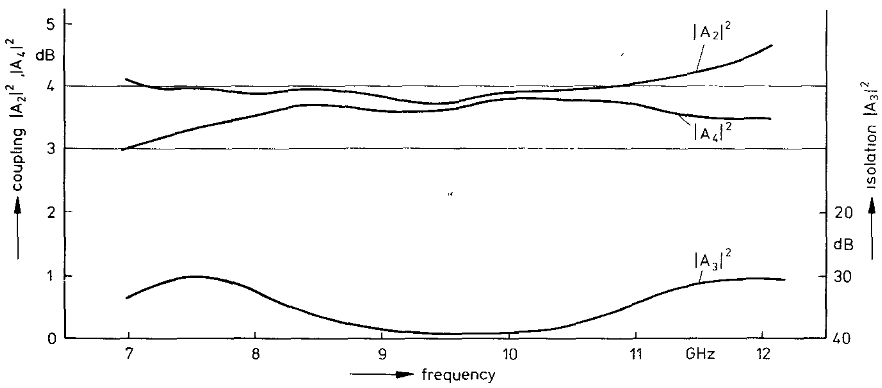

Fig. 13. Measured performance of an X-band coupler of the de Ronde type. Substrate thickness: 0.38 mm. Slot width: 50 μm. $\epsilon_r = 12$. Substrate: 2 × 2 cm.

broad-band strip-slot transition, if the coupled ports are, e.g., open-ended.

V. EXPERIMENTAL RESULTS

It is assumed that a slot crossing a microstrip simply represents a series impedance to the microstrip, at least to a first approximation.

For the design of a hybrid branchline coupler one has to know the value of this equivalent series impedance R_s presented to the microstrip. The slot is open-circuited on one side and matched on the other side (Fig. 11). The equivalent series impedance was measured in C band with the structure of Fig. 11 and different slot widths and was found to be nearly independent of frequency in the range 4–8 GHz. The substrate was nonmagnetic ferrite material, with a relative dielectric constant of 12.0 and a thickness of 0.63 mm. The microstrip had an impedance of 50 Ω. The impedance R_s was determined by measuring the increase in transmission loss when the slot was either present or short-circuited. The results, which are shown in Fig. 12, correspond to those given by Cohn [7] for the characteristic impedance of a slotline.

The impedance R_s is reduced further if the slotline is additionally loaded by a thick dielectric material with $\epsilon_r = 12$, as shown by the dotted curve in Fig. 12.

With the empirical knowledge of the slot impedance, a number of couplers of the de Ronde type (Fig. 1) have been constructed for C band (substrate thickness 0.63 mm,

$\epsilon_r = 12$) and X band (0.63 mm and 0.38 mm, and $\epsilon_r = 12$). Typical results for an X-band coupler, including the connectors, are shown in Fig. 13. When measuring the isolation, i.e., $|A_3|^2$ the effect of unwanted reflections from connectors has been reduced by putting absorbing material on the strips of the arms 2 and 4. Similar results but with total losses of about 0.3 dB have been obtained for C band. The strength of the coupling can be increased continuously (by about 1 dB) by loading the slot with dielectric materials of different thickness. The coupler performance was reproducible from sample to sample within a few tenths of a decibel and no correction in the design was necessary. No change in the coupler performance was noticed and no resonances occurred when a shielding box was used, leading to the conclusion that radiation loss is negligible. A balanced mixer made with such a coupler had a very constant noise figure of about 9–9.5 dB from 7–12.4 GHz.

Also an octave wide C-band matched leveller with two p-i-n diodes was built around this coupler with about 1-dB insertion loss and a minimum of 15-dB variable attenuation.

The slot-strip-slot-type coupler (Fig. 6) has also been realized in C band with a geometry similar to Fig. 4 but with slot and strip interchanged. With a slot width of 40 μm after etching and a dielectric loading, a coupling of 4 dB was obtained. The bandwidth was somewhat lower than for the de Ronde type of coupler and additional work is needed to optimize also the slot-strip-slot coupler.

VI. CONCLUSION

The hybrid branchline couplers with series and shunt branches are analyzed and shown to form a class of useful couplers, to which also the transmission-line coupler and de Ronde's strip-slot coupler belong. From the precise design data now available, a number of strip-slot 3-dB couplers have been built and their relevant parameters were close to the predicted ones.

ACKNOWLEDGMENT

The author wishes to thank W. Schilz, M. Lemke, J. Köhler, and F. C. de Ronde for valuable comments and discussions, and Mrs. H. Runge for help with the measurements and the numerical calculations.

REFERENCES

[1] J. Lange, "Interdigitated strip-line quadrature hybrid," in *1969 G-MTT Int. Microwave Symp.*, pp. 10–13.
[2] L. Young, "Parallel coupled lines and directional couplers," in *ARTECH House Reprint Volume.* Dedham, Mass.: ARTECH.
[3] F. C. de Ronde, "A new class of microstrip directional couplers," in *1970 IEEE Int. Microwave Symp.*, May 1970, pp. 184–186.
[4] J. A. Garcia, "A wide-band quadrature hybrid coupler," *Trans. Microwave Theory Tech.* (*Special Issue on Microwave Integrated Circuits*), vol. MTT-19, pp. 660–661, July 1971.
[5] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," *IRE Trans. Microwave Theory Tech.* (*Special Issue on the National Symposium on Microwave Techniques*), vol. MTT-4, pp. 246–252, Oct. 1956.
[6] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.
[7] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.

The Design of a Bandpass Filter with Inductive Strip—Planar Circuit Mounted in Waveguide

YOSHIHIRO KONISHI, SENIOR MEMBER, IEEE, AND KATSUAKI UENAKADA, MEMBER, IEEE

Abstract—**The equivalent circuit of an inductive strip inserted in the middle of a waveguide parallel to the E plane is analyzed theoretically by evaluating the inductive reactance of the equivalent T network which was obtained by the Rayleigh–Ritz variational technique. A design theory for the bandpass filter of this type is derived from this equivalent circuit. The confirmation between the design theory and the experimental results is also shown.**

Manuscript received January 22, 1974; revised April 17, 1974.
The authors are with the Technical Research Laboratories, Nippon Hoso Kyokai, Japan Broadcasting Corporation, Tokyo, Japan.

INTRODUCTION

FILTERS using conventional inductive elements such as rods, transverse strips, and transverse diaphragms are difficult to make low cost and to put into mass production because of their complicated structure. To solve this problem, microstrip circuits have been used. However, these, typically, are lossy. Therefore, we have developed a circuit consisting of a metal sheet with appropriate patterns that is inserted in the middle of a waveguide